

## Theme

*"Think deeply about simple things"*

### STEM Innovation Academy Unit 2

Subject: Integrated Math III

Unit Title: Building Functions and Inverse Functions

Grade: 10

Teacher: Ahmed Salama

Duration: 8 weeks

#### Summary of Unit

In this unit students will begin to explore radicals and rational functions. They will investigate the inverse relationship between radicals and exponential functions. In addition, students are going to explore polynomial functions with rational and integer exponents. They will be able to explore the features of radical and rational functions and compare their different functions by certain features such as end behavior, average rate of change, etc. Finally, students can determine the appropriate function to model a situation as well as investigate how transformations of functions relate to their parent function.

#### Stage 1 – Desired Results

##### Essential Questions:

- How can functions be manipulated to make new functions?
- What is the relationship between a function with its inverse?
- How to decide a domain for a inverse function in real-life situation?

##### Enduring Understandings:

- A rational function is a function formed by the quotient of two polynomials.
- Parent function of a rational function is  $1/x$
- Inverse proportion is when one value decreases at the same rate that the other increases.
- You can model a rational function with a table, equation, and graph.
- Corresponding to every power there is a root
- You can combine like radicals using properties of real numbers
- You can write a radical expression in an equivalent form using a fractional (rational) exponent instead of a radical sign
- Solving a square root equation may require that you square each side of the equation. This process can introduce extraneous solutions

##### Objectives:

By completing the unit, students will be able to

- apply transformations to graphs of parent functions
- recognize the general form of a quadratic equations and explains how the values of a, h, and k affects the shape of the parabola
- describe the transformations from one function to another in terms of vertical shifts, vertical shrink, stretches, and horizontal shifts
- relate geometric transformations to tables of values for functions
- understand graphical, tabular, and algebraic relationship between a linear function and its inverse.
- Understand the relationship between quadratic and square root functions
- Identify one-to-one functions
- restrict the domain of a quadratic function for its inverse to be a function

- solve radical equation and graphically and algebraically. Identify extraneous solution and in which step caused extraneous solution
- define rational function and create rational functions.
- Interpret models of rational functions.
- demonstrate transformation of functions on rational functions using parameter changes.

### Standards/Outcomes: NJSL

- APR D.6: Rewrite rational expressions 6. Rewrite simple rational expressions in different forms; write  $a(x)/b(x)$  in the form  $q(x) + r(x)/b(x)$ , where  $a(x)$ ,  $b(x)$ ,  $q(x)$ , and  $r(x)$  are polynomials with the degree of  $r(x)$  less than the degree of  $b(x)$ , using inspection, long division, or, for the more complicated examples, a computer algebra system.
- A.APR.D.7: (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.
- A-CED A.2: equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
- F-IF A.1: Understand the concept of a function and use function notation 1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ . B. Interpret functions that arise in applications in terms of the context
- F-IF B.4: Interpret functions that arise in applications in terms of the context. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.
- F-IF B.5: Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function  $h(n)$  gives the number of person-hours it takes to assemble  $n$  engines in a factory, then the positive integers would be an appropriate domain for the function.
- F-IF.7b: Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
- F-BF B.3 Build new functions from existing functions 3. Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $k f(x)$ ,  $f(kx)$ , and  $f(x + k)$  for specific values of  $k$  (both positive and negative); find the value of  $k$  given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.
- A-CED. A.1: Create equations that describe numbers or relationships 1. Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.
- A-CED.A3: Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. For example,

represent inequalities describing nutritional and cost constraints on combinations of different foods.  
Reasoning with Equations and Inequalities

- A-REI A.2: Understand solving equations as a process of reasoning and explain the reasoning 2. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.
- 1) A.RE.D.11. Explain why the x-coordinates of the points where the graphs of the equations  $y = f(x)$  and  $y = g(x)$  intersect are the solutions of the equation  $f(x) = g(x)$ ; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where  $f(x)$  and/or  $g(x)$  are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.

**Unit Math Practice Standards:**

- **MP.1** Make sense of problems and persevere in solving them.
- **MP.2** Reason abstractly and quantitatively.
- **MP.3** Construct viable arguments and critique the reasoning of others.
- **MP.4** Model with mathematics.
- **MP.5** Use appropriate tools strategically.
- **MP.6** Attend to precision.
- **MP.7** Look for and make use of structure.
- **MP.8** Look for and express regularity in repeated reasoning.

**Stage 2 – Assessment Evidence**

**Performance Task(s):**

- Task 1: Who wins the race
- .Odd & Even Functions

**Authentic Experiences:**

- ‘Use the concept of inverse to develop the formula to convert temperature between Fahrenheit and Celsius
- Use the concept of inverse to create formula/equation to find unknown number in real-life setting

**Unit Pre-Assessment:**

- Unit 2 Readiness Assessment
- NWEA Diagnostic Assessment
- Benchmark 1 Assessment

**Summative Assessment:**

- District created Benchmark 2 Assessment

**Extensions (Tier I):**

- Enrichment Question (challenging add-ons)
- More applications of findings

**Differentiation (Tiers 2 and 3):**

- Selective grouping
- Extended time
- Small groups / Individual instruction
- Use Bloom’s taxonomy framework to develop multiple-choice, short-answer, matching, and essay question that could lead.
- Use online assessment in our teaching in several ways

## Stage 3 – Learning Plan

### Vocabulary

- Transformation of functions
- Parent functions
- Vertical shift
- Horizontal shift
- Vertical stretch
- Vertical shrink,
- Vertical compressions
- Even and Odd functions
- Point of symmetry
- Line of symmetry
- Inverse functions
- Parabola
- Rational functions
- Radical functions

### Learning Materials:

Textbook: CPM Core Connections Algebra 2

Section 2.2.1: How can I transform any graph? Transforming Other Parent Graphs (Chapter 2, Online 2-76 -- 2-101)

Section 2.2.2: What is the significance of  $(h, k)$ ? Describing  $(h, k)$  for Each Family of Functions (Chapter 2, Online 2-102 -- 2-120)

Section 2.2.3: How can I move a function? Transformations of Functions (Chapter 2, Online 2-121 – 2-131)

Section 3.2.1: Where does the graph go? Investigating Rational Functions (Chapter 3, Online 3-57 – 3-69)

Section 3.2.2: How can “1” be useful? Simplifying Rational Expressions (Chapter 3, Online 3-70 – 3-84)

Section 3.2.3: How can I rewrite it? Multiplying and Dividing Rational Expressions (Chapter 3, Online 3-85 – 3-96)

Section 3.2.4: How Can I rewrite it? Adding and Subtracting Rational Expression (Chapter 3, Online 3-97 – 3-109)

Section 3.2.5: Pulling it all together. Creating New Functions (Chapter 3, Online 3-110 – 3-126)

Section 5.1.1: How can I “undo” a function? Undo Equations (Chapter 5, Online 5-1 – 5-15)

Section 5.1.2: How can I find an inverse? Using a Graph to Find an Inverse (Chapter 5, Online 5-16 – 5-39)

Section 5.1.3: What can I do with inverses? Finding Inverses and Justifying Algebraically (Chapter 5, Online 5-40 – 5-54)